ТОВ «ТЕХНІЧНИЙ УНІВЕРСИТЕТ
«МЕТІНВЕСТ ПОЛІТЕХНІКА»

RISK ANALYSIS AND MANAGEMENT

**methodological recommendations
for independent work**

Запоріжжя 2024

metinvest polytechnic

УДК 330.131(072)

А64

Рекомендовано Науково-методичною радою
ТОВ «ТЕХНІЧНИЙ УНІВЕРСИТЕТ
«МЕТІНВЕСТ ПОЛІТЕХНІКА»
(протокол №3 від 24.01.2024 р.)

Укладач

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А64 **Risk Analysis and Management** : Methodological Recommendations for Independent Work / уклад. Н. В. Рагуліна. Запоріжжя : ТОВ «ТЕХНІЧНИЙ УНІВЕРСИТЕТ «МЕТІНВЕСТ ПОЛІТЕХНІКА», 2024. 26 с.

The methodological recommendations provide theoretical and practical aspects of independent work in the discipline ‘Risk Analysis and Management’: historical and theoretical information on game theory and the construction of mathematical models.

УДК 330.131(072)

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Introduction

Formalising the process of risk calculation using game theory helps entrepreneurs to better understand the problems they face in general. Thus, game theory is a science that studies risks and contributes to the solution of numerous economic problems related to the selection of optimal (best) solutions that meet the constraints determined by the conditions of a particular problem. The methodological recommendations for independent work in the discipline ‘Risk Analysis and Management’ provide theoretical foundations and practical opportunities for modelling economic security through the prism of game theory. New opportunities that open up when modelling economic security processes are considered separately, approaches and stages of solution search are presented, supported by examples of effective use and emphasising the importance of game theory tools. The modelling process is considered step by step, indicating the general and specific requirements that are characteristic of the entire model of the consequences of management decisions in the field of economic security of the organisation.

After studying the discipline, taking into account the mastery of this material, students will gain a comprehensive understanding of risk management processes and acquire the skills to use the knowledge gained to make effective decisions in real life.

1. HISTORICAL BACKGROUND

The development of game theory was based on the search for optimal solutions (strategies), which began to be developed as part of mathematical modelling in the 18th century. The problems of production in an oligopoly, considered by the French mathematicians and economists Antoine Cournot and Joseph Bertrand in the 19th century, can be considered practical examples of game theory. Later, the Hungarian mathematician Abraham Wald proposed some fundamental ideas of a new approach to statistical decision theory. However, all of these models of decision-making in economic systems considered participants as being focused on increasing their own profits and not taking into account the activities of other participants in the economic system when making their decisions. Such assumptions ignored competition as one of the main factors influencing the behaviour of market participants. By studying various economic models, scientists came to the conclusion that the activity of a participant in the economic system most closely resembles a game against other players. This led them to the idea of treating an economic model as a special case of a game, and its participants competing with each other as players. The mathematicians-economists John von Neumann (officially recognised as the founder of game theory) and Oscar Morgenstern proposed to remedy this situation. They developed an economic model as a special case of a game where participants act as players and compete with each other. In 1944 these scientists published their work “Game Theory and Economic Behaviour”, which formulated the following:

- a definition of a “game” as an activity of two or more participants that has certain conditions of ‘winning’ and “losing”, where participants manage resources, interact with each other and make decisions, taking into account the behaviour of other players;
- a mathematical method for finding optimal game strategies that ensure ‘winning’ with a certain probability.

The game theory was further developed by the American mathematician John Forbes Nash, who in 1949, at the age of 21, wrote a dissertation on game theory, for which he later received the Nobel Prize in Economics. In his work, he developed an equilibrium formula (later called the ‘Nash equilibrium’ or ‘non-cooperative equilibrium’) that describes a set of strategies in which each participant implements an optimal strategy by anticipating the actions of rivals. Game theory has been applied not only in economics (to model relationships between suppliers and consumers, buyers and sellers, banks and clients), but also in military affairs, where it has been used since the Second World War to study strategic decisions. Thus, the interest in game theory methods is not waning even today [1].

2. THEORETICAL INFORMATION

Game theory is a branch of mathematical economics that studies the possibility of resolving conflicts between players and the optimality of their strategies. For each player, there is a certain set of strategies that he can apply. When the strategies of several players intersect, they form a certain situation in which each player receives a certain outcome, which is called a winning, positive or negative outcome.

Game theory is the theory of mathematical models in which the interests of the participants are different and can be achieved in different ways.

The essence of game theory is to establish the optimal (in one sense or another) strategy for a player's behaviour in conflict situations.

The goal of game theory is to predict the outcomes of strategic, operational games when participants do not have full information about each other's intentions.

A simplified model of a conflict situation is called a game. At the same time the game is understood as a certain process consisting of a series of actions or “moves”. To solve a game means to find the price of the game and the optimal strategies of the players. The rules of the game determine the possible options for players' actions, the amount of information each party has about the actions of the other, and the outcome of the game that a particular sequence of moves will lead to.

In most games, it is assumed that the interests of the participants can be quantified, i.e. the outcome of the game (winning) is determined by a certain number.

In theoretical terms, the course of a game is the choice of one of the actions allowed by the rules of the game and its implementation. The main thing in the game model is that the other party - the opponent - actively opposes the player in choosing the optimal solution, and therefore it is necessary to objectively assess his or her activities.

The game differs from a real conflict situation in that it is conducted according to certain rules.

The parties involved in the conflict are called ‘players’, and the result of the ‘conflict’ is called “winning”.

A conflict is a situation in which the interests of two or more parties with different (sometimes opposing) goals clash.

The classification of games is determined according to the selected criteria. Games are distinguished depending on the number of players, the number of strategies, the properties of winning functions, the possibilities of interaction between players, etc.

Depending on the number of players, games are divided into pairs (two players take part) and multiplayer (more than two players take part).

Depending on the number of strategies, there are finite and infinite games. In finite games, the number of possible strategies is a finite number (two strategies for a coin toss, six strategies for a dice roll), while in infinite games, the number of strategies is infinite. Strategies in finite games are called pure strategies.

Depending on the restrictions on the amount of winnings, we distinguish between zero-sum games and games with an arbitrary amount. If one player's win is equal to the other's loss, the game is called a zero-sum game. Such games are characterised by opposing interests of the parties, and the gain of one player of a certain amount means that another player (a set of other players) loses this amount, which determines the situation of conflict. Therefore, such games are often called antagonistic.

According to the type of payoff functions, games are divided into matrix, bimatrix, continuous, convex, and others.

For matrix games, it is proved that any of them has a solution that can be found by reducing the game to a linear programming problem.

A matrix game is a finite two-player zero-sum game in which the payoff of player 1 is given in the form of a matrix (the row of the matrix corresponds to the number of player 1’s strategy, which is used, and the column corresponds to the number of player 2’s chosen strategy; the intersection of the row and column of the matrix contains the payoff of player 1, which corresponds to the strategies chosen by the players). In cases where the problem is reduced to a matrix form, one can ask about finding optimal strategies. To do this, we need to consider the concepts of upper and lower game prices.

The lower game price shows that no matter what strategy player B uses, player A guarantees himself a payoff no less than A's.

The upper game price guarantees for player B that player A will not receive a payoff greater than β.

The solution of the matrix game is the saddle point. At this point, the largest of the minimum winnings of player A is exactly equal to the smallest of the maximum losses of player B, i.e. the minimum in any row of the matrix coincides with the maximum in any column.

When analysing the payment matrix, there are two possible cases of choice evaluation:

1) the payment matrix has a saddle point. Since we have assumed maximum rationality of the players, these rows and columns represent the optimal strategies of the players. If one of the players uses the optimal strategy, then it is disadvantageous for the other player to deviate from his optimal strategy. That is, strategies that correspond to the saddle point are the most profitable for both players;

2) the payment matrix does not have a saddle point. This is the most common case, for which it is proposed to be guided by the so-called mixed strategies, i.e. those strategies in which personal strategies are randomly alternated.

A bimatrix game is a finite two-player game with a non-zero sum, in which the payoffs of each player are given by matrices separately for the respective player (in each matrix, a row corresponds to the strategy of player 1, a column to the strategy of player 2, the intersection of a row and a column in the first matrix contains the payoff of player 1, and the second matrix contains the payoff of player 2).

A continuous game is a game in which the payoff function of each player is continuous. It has been proven that games of this class have solutions, but no acceptable methods for finding them have been developed to date.

In the economy, situations sometimes arise when the effectiveness of a decision by one participant depends on decisions made by other participants. Such situations are called conflicts. Game theory is concerned with building mathematical models to analyse conflict situations and develop methods for solving problems that arise in these situations.

One of the peculiarities of conflict situations is that players can act not only consciously but also randomly. For example, in a game with nature, one of the participants, player 1, makes a decision, while the other participant, nature, acts randomly and has no specific goal. In a game with nature, player 1 makes a decision, and nature randomly chooses its next move. This game is used to analyse situations where players cannot fully control their actions or do not know all the parameters of the game. In such cases, player 1 has to choose a strategy that allows him to maximise his winnings under any possible moves of nature. Thus, game theory is an important tool for analysing conflict situations in the economy and can be used to make decisions in various fields such as business, politics and science.

The term “nature” in the context of game theory means objective reality, but it should not be taken literally. Some situations may be related to real natural factors, such as weather conditions or natural disasters, but in most cases nature is an abstract concept.

In some cases, nature can be interpreted as a competitive environment, such as a securities market that is opposed to the investor. In such cases, knowledge of the optimal strategies of nature helps player 1 to determine the most favourable conditions that may await him in the future and to estimate the necessary resources to achieve his goals.

The application of game theory methods to portfolio analysis allows us to consider the interaction between various assets and investors as a game in which each participant seeks to maximise its own benefits, subject to limited resources and competition from other participants. Using this approach, we can identify optimal strategies for building an investment portfolio and predict the behaviour of other market participants.

To build an investment portfolio, an investor should consider his or her goals and attitude to risk. The goals may be different: generating a stable income, protecting against inflation, maximising returns, etc. The attitude to risk can be conservative, aggressive or moderate, depending on how much risk the investor is willing to take with investments. The choice of assets for investment should also be based on the goals and attitude to risk. An investor can choose different types of assets such as stocks, bonds, funds, etc. It is important to take into account the characteristics of each asset, such as return, risk, liquidity, etc.

When building an investment portfolio, it is also necessary to take into account different investment periods. An investor can choose short-term, medium-term or long-term investments depending on his or her goals and attitude to risk. Portfolio investing allows an investor to plan, evaluate and control the results of his or her investment activities. The purpose of portfolio investing is to improve investment conditions by combining different assets to obtain the most favourable result for the investor. Game theory methods can be used to analyse the actions of an investor who seeks to obtain the most favourable result when investing in securities. Game theory helps to analyse such actions in the context of the investor's existing portfolios.

By applying the ‘Playing with Nature’ method to investing in securities on the Ukrainian market, you can determine the optimal strategies for investors. For this purpose, portfolios are analysed that have minimal risk for different types of investors. Out of all the efficient portfolio sets, the ones that are considered optimal according to these criteria are selected. Thus, investment and portfolio game theory can be useful in determining optimal investment strategies for securities on the stock market. The analysis of portfolios with minimum risk for different types of investors allows us to select the most efficient portfolios that provide maximum return with minimum risk.

Let's assume that nature acts in contrast to the interests of the investor and tries to harm him as much as possible. Based on this assumption, you can estimate the investor's return if time plays on the side of nature.

Investors can use three pure strategies to do this:

1. Aggressive portfolio. An investor plays the securities market with an aggressive portfolio, which can provide high returns but also carries high risks.

2. Balanced portfolio. The investor operates with a balanced portfolio, which implies a more conservative approach and risk reduction, but may also provide lower returns.

3. Passive portfolio. The investor builds only a passive portfolio that does not require active management and can provide a stable, but not high income.

Using these three strategies, an investor can estimate the possible return under conditions when nature tries to harm his interests as much as possible. If the game does not have a point at which it is possible to achieve maximum gain for all players, then several pure strategies should be used to increase the income, not just one. In other words, a player should alternate between different pure strategies of each game to maximise his or her winnings [3]. An investor should not limit his choice to only one portfolio for a certain period. This is because the ability to use all available options for securities portfolios contributes to higher returns by following optimal mixed strategies. If the assets included in the portfolios increase in value over the period under consideration, the chosen plan will increase income. The optimal decision-making criterion is to maximise the mathematical expectation of gain based on the known probability distribution of different states of nature.

It can be concluded that different game theory methods lead to almost identical recommendations for investor actions. However, there is a difference in the use of mixed strategies: in one case, all three optimal portfolios can be used, while in the other, only one of the most profitable ones can be used. When choosing a mixed strategy, it is more profitable for an investor to choose strategies based on investment periods. As a result, an aggressive portfolio is the most profitable for short-term investments, a balanced portfolio for medium-term investments, and a passive portfolio for long-term investments.

Today, investors often use portfolio analysis methods to form an optimal investment portfolio, choosing assets of Ukrainian issuers depending on the type of portfolio: aggressive, balanced or passive. Portfolio returns and risk vary depending on the term of investment in assets, which can both increase the investor's return and cause losses [5].

Although it is impossible to accurately predict the behaviour of assets on the securities market at a given time due to the many factors that influence the dynamics of listed issuers, it is beneficial for investors to build an aggressive portfolio for the short term, a balanced portfolio for the medium term and a passive portfolio for the long term. To determine the optimal strategies and maximise the guaranteed return, investors can use game theory methods instead of trying to predict the behaviour of assets in the securities market. This allows the investor to achieve maximum portfolio returns regardless of the investment period used.

Using game theory methods, forecasting changes in the securities market helps investors determine the optimal strategy for different time periods, which helps to increase the expected return in the face of risk and uncertainty. Game theory has been used to develop a methodology that allows us to explain previously unexplained phenomena and take into account asymmetric information and strategic interaction in our analysis.

The most common games are doubles, which we will consider. Let's denote the participants of the game as A and B. A game is a certain sequence of actions (moves) of players A and B, which takes place according to clearly defined rules. In most games, it is assumed that the interests of the players can be described quantitatively, i.e. the outcome of the game (winning) is determined by a certain numerical value. A player's strategy is a plan according to which he or she makes a choice in any situation and with any available information. The goal of game theory is to create recommendations for players, i.e. to determine the optimal strategy for them. The optimal strategy is the one that provides the player with the maximum average payoff when the game is repeated many times. Now let's create the matrix A:

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$$

Rows of the matrix correspond to strategies A_i, columns to strategies B_j. Matrix A is called the payoff matrix or game matrix. Each element aij of this matrix represents the payoff of player A if he chooses strategy A_i, and player B - strategy B_j. In this case, each player seeks to maximise his or her payoff. Numerical values in the matrix can be either positive, negative or zero. If a_{ij} >0 , it corresponds to a win for the j-th player, if a_{ij} <0 , it means a loss, and if $a_{ij} = 0$ - a draw. In most cases, we have zero-sum games $\sum_{j=1}^{n} a_{ij} = o$, *i=1,...,m*. In such games, the total payoff is simply redistributed among the players without external funding. A zero-sum game means that the sum of the winnings of all participants in each round is zero. Many economic problems are examples of such games, where the winnings are only distributed among the players, but the total amount does not change. Otherwise, we are dealing with non-zero-sum games.

Suppose that player A chooses strategy A_i; then in the worst case (e.g., if the choice is known to player B), he will receive a payoff equal to *min* a_{ij}. Foreseeing this situation, player A should choose the following strategy to maximise his minimum payoff α : $\alpha = \max \min a_{ij}$. The value α - is the guaranteed payoff of player A and is called the lower price of the game. The strategy A_{i0}, that ensures obtaining α , is called the maximal strategy.

Player B, when choosing his strategy, is guided by the following principle: when choosing strategy B_i his loss will not exceed the maximum value among the elements of the corresponding column of the matrix, i.e. it will be less than or equal to the value *max* a_{ij}. Considering the set of *max* a_{ij} for different outcomes *j,* the player naturally chooses the value of *j*, at which his maximum loss β is minimised: $\beta = minmax\, a_{ij}$.

The value β is called the upper price and corresponding to the gain β strategy β_{j0} is called the minimum strategy. In case, if $\nu = \alpha = \beta$, then the matrix game is called a saddle point game, and the value of ν is called the game price. The saddle point corresponds to the optimal strategies of both players, and their set is the solution of the game, which has the following property: if one of the players chooses his optimal strategy, then the other player does not benefit from deviating from this strategy. Therefore, if the game has a saddle point, the solution of the game is determined. However, the question arises as to how to find solutions for games whose matrices do not have a saddle point. In these games $\alpha < \beta$. The way to find a solution is that players use not one, but several strategies, choosing them randomly. Such a random choice of strategies by a player is called a mixed strategy.

In a game with a matrix of dimension $m \times n$, player A's strategies are given by the probability sets $X=(x_1,x_2,\ldots,x_m)$, with which the player applies his initial pure strategies, whereby $\sum_{i=1}^{m} x_i = 1, x_i \geq 0$. Similarly, the strategies of player

B are defined by the probability set Y=(y1,y2,…,yn), $\sum_{i=1}^{n} y_i = 1, y_i \geq 0$. The main theorem of game theory states that every finite game has at least one solution that can be found within the framework of mixed strategies. Using the optimal strategy allows you to get a payoff equal to the price of the game: $\alpha \leq \nu \leq \beta$.

The task of solving a game, if its matrix does not contain a saddle point, becomes more difficult as the values of *m* and *n* increase. That is why the theory of matrix games considers methods that allow to reduce the solution of some games to the solution of other, simpler ones (in particular, by reducing the matrix dimension). The matrix dimension can be reduced by eliminating duplicate strategies and obviously disadvantageous dominant strategies.

Duplicate strategies are those that have the same values of the elements of the payment matrix, i.e., the matrix contains the same rows or columns. If all the elements of the i-th row of the matrix are smaller than the corresponding elements of the k-th row, then the i-th strategy for player A is called dominant. If the elements of the r-th column of the matrix are greater than the corresponding elements of the j-th column, then the strategy B_r for player B is dominant.

So, when solving the game $m \times n$, it is necessary:

1) check whether there is a saddle point in the matrix;

2) if there is no saddle point, compare the elements of the rows and columns to eliminate duplicate and dominant strategies.

Thus, it should be concluded that there are currently many ways to assess risks, and new ones will appear in the future, but at the same time, game theory models are among the most effective means of risk assessment. Using mathematical methods and a specialised approach to risk assessment, game theory allows us to find optimal strategies in various fields, including its use for decision-making in conditions of uncertainty and risk in financial markets. Game theory is a complementary tool for analysing assets and should not be used as the sole criterion. For a more complete analysis, it is necessary to combine fundamental, technical and game theory analysis.

Risk assessment criteria

The main criteria used to select a solution with minimal risk (optimal solution) are the Laplace, Wald, Savage, and Hurwitz criteria. Let's consider the application of these game theory criteria in risk assessment using the example of a single player: The director of the trading company Apple, plans to release a batch of new phones, he can place all the new goods in 5 outlets: A_1, A_2, A_3, A_4, A_5. The company's success largely depends on how the situation on the service market develops. Experts identify 4 possible scenarios: A_1, A_2, A_3, A_4. Estimates of the company's profit in each situation are represented by the winning matrix A (UAH million per year). The initial data on possible winning options are presented in Table 1.

Table 1

Initial data on possible winning options

Retail outlets	Scenarios			
	B_1	B_2	B_3	B_4
A_1	6	7	5	7
A_2	7	6	4	5
A_3	8	6	7	8
A_4	7	8	6	5
A_5	3	8	5	9

Let us consider the main criteria that allow you to choose the best alternative for making a decision. According to Laplace's criteria, which are based on the assumption that each scenario is equally likely to occur. Therefore, to make a decision, it is necessary to calculate the utility function A for each alternative. The alternative with the highest utility function is chosen.

$$A_1 = \frac{1}{4} * (6 + 7 + 5 + 7) = 6{,}25;$$
$$A_2 = \frac{1}{4} * (7 + 6 + 4 + 5) = 5{,}5;$$
$$A_3 = \frac{1}{4} * (8 + 6 + 7 + 8) = 7{,}25;$$
$$A_4 = \frac{1}{4} * (7 + 8 + 6 + 5) = 6{,}5;$$
$$A_5 = \frac{1}{4} * (3 + 8 + 5 + 9) = 6{,}25.$$

It can be seen that the utility function is maximised for alternative A_3, which means that A_3 can generate the highest profit from the outlets, and therefore alternative A_3 is the most rational to adopt. Wald's criterion is based on the assumption that the worst-case scenario is most likely to occur.

Therefore, you need to select the smallest number in each row of the payoff matrix and choose the alternative for which this indicator is maximum. For our example:

$A_1 = 5;$

$A_2 = 4;$

$A_3 = 6;$

$A_4 = 5;$

$A_5 = 3.$

It is easy to see that the utility function is maximised for alternative A_3, which means that A_3 can generate the most profit from the outlets, and therefore alternative A_3 is the most rational to adopt. Wald's Criterion is based on the assumption that the worst-case scenario is most likely to occur. Therefore, you need to select the smallest number in each row of the payoff matrix and choose the alternative for which this indicator is maximum. For our example:

Table 2

Savage Criteria Loss Matrix

	B_1	B_2	B_3	B_4
A_1	2	1	2	2
A_2	1	2	3	4
A_3	0	2	0	1
A_4	1	0	1	4
A_5	5	0	2	0

Next, for each alternative we determine the values of B equal to the maximum risk and choose the alternative for which the maximum risk is minimal. In the example:

$A_1 = 2;$

$A_2 = 4;$

$A_3 = 2;$

$A_4 = 4;$

$A_5 = 5.$

(minimum $A_1 = 2$, $A_3 = 2$).

It is necessary to make the decision $A_1 = 2$ or $A_3 = 2$, since in this option the deviation of the risk from the minimum is the smallest of the maximum, i.e. in outlets A_1 and A_3, the possible loss of profit will be UAH 2 million per year, which is the smallest of the maximum possible losses in the event of a risk event. The Hurwitz Criterion is the most versatile method that allows managing the degree of "optimism - pessimism" of the company (player A). Let us introduce a certain coefficient α, which we will call the optimism coefficient, which shows the probability with which the best outcome for the company will occur. Based on this, the worst case scenario is expected with a probability of (1-α). The confidence factor shows the extent to which the company can manage the situation. Let's assume that in our example, the company is confident enough in a positive outcome to estimate the probability of maximum success at α = 0,6. According to the information in Table 1, we have:

$$A_1 = 7 * 0,6 + 5 * (1 – 0,6) = 6,2;$$
$$A_2 = 7 * 0,6 + 4 * 0,4 = 5,8;$$
$$A_3 = 8 * 0,6 + 6 * 0,4 = 7,2;$$
$$A_4 = 8 * 0,6 + 5 * 0,4 = 6,8;$$
$$A_5 = 9 * 0,6 + 3 * 0,4 = 6,6.$$

Based on the results of the example analysis, the company (player A) should choose alternative A_3, i.e. the lowest risk that the batch of nothing is sold at outlet A_3.

3. BUILDING MATHEMATICAL MODELS

The simplest matrix game is a game in which each player has two strategies. The matrix A of the game has the form:

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}.$$

If there is no saddle point, then the game is solved by mixed strategies X = (x_1, x_2), Y = (y_1, y_2). According to the basic theorem of game theory, the use of the optimal strategy X = (x_1, x_2) ensures that player A wins under any strategy of player B. The optimal strategy for a player is also mixed. Therefore, if player A chooses his optimal strategy, he can use one of the pure strategies, and the value of his winnings will remain unchanged.

Let's write the system of equations

$$\begin{cases} a_{11}x_1 + a_{21}x_2 = \nu \\ a_{12}x_1 + a_{22}x_2 = \nu' \end{cases}$$

since $x_1+x_2=1$, the solution has the following form:

$$x_1 = \frac{a_{22} - a_{21}}{a_{11} + a_{22} - a_{12} - a_{21}}; x_2 = \frac{a_{11} - a_{12}}{a_{11} + a_{22} - a_{12} - a_{21}};$$

$$\nu = \frac{a_{11}a_{22} - a_{12}a_{21}}{a_{11} + a_{22} - a_{12} - a_{21}}.$$

By composing a similar system of equations, we can find the optimal strategy for player B:

$$y_1 = \frac{a_{22} - a_{12}}{a_{11} + a_{22} - a_{12} - a_{21}}; y_2 = \frac{a_{11} - a_{21}}{a_{11} + a_{22} - a_{12} - a_{21}}.$$

Let us consider the application of mathematical models of game theory on specific examples.

Task 1: Find the solution of the game given by the matrix: $A = \begin{pmatrix} 1 & 3 \\ 2 & 1 \end{pmatrix}$

<u>*Answer:*</u> Find the lower and upper price of the game:

$$\alpha = \max \min a_{ij} = \max(1,1) = 1,$$

$$\beta = \min \max a_{ij} = \min(2,3) = 2,$$

i.e. $\alpha = 1, \beta = 2$, therefore, the matrix has no saddle point.

By the formulas, we have the optimal strategies and the price of the game:

$$x_1 = \frac{1-2}{1+1-3-2} = \frac{-1}{-3} = \frac{1}{3}; \quad x_2 = \frac{1-3}{1+1-3-2} = \frac{-2}{-3} = \frac{2}{3}; \quad \nu = \frac{1*1-3*2}{1+1-3-2} = \frac{-5}{-3} = \frac{5}{3};$$

$$y_1 = \frac{1-3}{1+1-3-2} = \frac{-2}{-3} = \frac{2}{3}; \quad y_2 = \frac{1-2}{1+1-3-2} = \frac{-1}{-3} = \frac{1}{3}.$$

Or finally in the form:

$$X = \left(\frac{1}{3}, \frac{2}{3}\right), \ Y = \left(\frac{2}{3}, \frac{1}{3}\right), \ \nu = \frac{5}{3} = 1\frac{2}{3}.$$

Task 2: Consider two competing companies A and B that produce the same type I and type II products made of two materials: plastic or aluminium. A study of consumer demand has shown that if type I or type II products are made of plastic, 40% of buyers choose type I plastic and the remaining 60% choose type II plastic. If the products are made of type I plastic and type II aluminium, the consumer chooses type I plastic with a 90% probability. If type I products are made of aluminium and type II products are made of plastic, then 70% of consumers will choose aluminium products. If both type I and type II are made of aluminium, then 20% of buyers will choose type I.

Construct a matrix game and find the optimal strategies of the players and the price of the game.

Answer: Let us draw up a game matrix from the position of player A.

Table 3.

Matrix game problem 2.

A \ B	II plastic	II aluminum	α_i
I plastic	-20	80	-20
I aluminum	40	-60	-60
β_j	40	80	-

Let's find the lower and upper price of the game:

$$\alpha = \max \min a_{ij} = \max(-20, -60) = -20,$$

$$\beta = \min \max a_{ij} = \min(40, 80) = 40,$$

i.e. $\alpha = -20, \beta = 40$, therefore, the matrix has no saddle point.

According to the formulas, we have the optimal strategies and the price of the game:

$$x_1 = \frac{-60-40}{-20-60-80-40} = \frac{-100}{-200} = \frac{1}{2};$$

$$x_2 = \frac{-20-80}{-20-60-80-40} = \frac{-100}{-200} = \frac{1}{2};$$

$$v = \frac{-20*(-60)-40*80}{-20-60-80-40} = \frac{-2000}{-200} = 10;$$

$$y_1 = \frac{-60-80}{-20-60-80-40} = \frac{-140}{-200} = \frac{7}{10}; y_2 = \frac{-20-40}{-20-60-80-40} = \frac{-60}{-200} = \frac{3}{10}.$$

Or in its final form:

$$X = \left(\frac{1}{2}, \frac{1}{2}\right),\ Y = \left(\frac{7}{10}, \frac{3}{10}\right),\ v = 10\ .$$

Thus, under optimal strategies, Company A should produce products from plastic and aluminum in equal proportions, and Company B should produce 70% of its products from plastic and 30% from aluminum. In this case, Company A wins and its profit is 10 units.

Task 3: Two rabbits, one large and one small, are placed in a room with a lever at one end and a feeder at the other end. To get food, one of the rabbits has to push the lever. When the rabbit presses the lever, the dispenser feeds 12 units of food into the trough. If the large rabbit reaches the feeder first, the small rabbit will only receive the equivalent of 2 units of food. If the small rabbit gets to the trough first, it can eat 6 units. If both rabbits are at the trough at the same time, the small rabbit will get 4 units. Make a win matrix for the big rabbit.

Answer: Let us designate the players: player A - a large rabbit, player B - a small rabbit and consider the players' strategies:

A1 - the large rabbit was at the feeder first;

A2 - the large rabbit was at the feeder at the same time as the small one;

A3 - the large rabbit was at the feeder later than the small one;

Q1 - the small rabbit reached the feeder first;

Q2 - the small rabbit was at the feeder at the same time as the large one;

Q3 - the small rabbit was at the feeder later than the large one.

Let us make a payoff matrix for the first player - the large rabbit.

Table 4.

Model for creating a solution to the matrix game for problem 3.

Big rabbit/ little rabbit	**issue 1**	**issue 2**	**issue 3**
Answer 1	The big rabbit came first, the little rabbit came first - the situation is impossible, so the win is 0	The big rabbit came first, and the small one came at the same time as the first one - the situation is impossible, so the win is 0	The big one came first, and the small one came second, then the second rabbit gets leftover food equal to 2 units, so the first rabbit will eat 12-2=10 units of food

Answer 2	The big rabbit will get to the feeder at the same time as the small one, and the small rabbit will get there before the big one - this situation is impossible, so the gain is 0.	The large rabbit will get to the feeder at the same time as the small one, and the small rabbit will get to the feeder at the same time as the large one, which means that the small one will get 4 units of food, so the first rabbit's gain is 12-4=8 units of food.	The big rabbit will get to the feeder at the same time as the small one, and the small rabbit will come second - the situation is impossible, so the feedback is 0
Answer 3	The big rabbit came second and the small one came first, which means the small rabbit eats 6 units of food, so the first one will eat 12-6=6 units of food.	The big pig came second, and the little pig came at the same time as the first - the situation is impossible, so the win is 0	The big pig came second, and the little pig also came second - the situation is impossible, so the win is 0

As a result, the payoff matrix takes the form (see Table 5).

Table 5.

Game matrix for problem 3.

Big rabbit/ little rabbit	**issue 1**	**issue 2**	**issue 3**
Answer 1	0	0	10
Answer 2	0	8	0
Answer 3	6	0	0

3.1. APPLICATION OF MS EXCEL

Let us consider the implementation of finding the optimal solution to a 2x2 matrix game using MS EXCEL. Figure 1 shows the solution to task 1, obtained by the third method.

	A	B	C	D	E
1	**Example.** For this game matrix, determine the optimal strategies of the players and the price of the		A=	1	3
2				2	1
3	**Answwer:**				
4	The answer of the game is the mixed strategies of players A and B.		X=	1/3	2/3
5					
6			Y=	2/3	1/3
7					
8	with the price of the game:		v=	12/3	

Figure 1. *Implementing the solution to problem 1 using MS EXCEL.*

Task 4. Let enterprises A and B produce ceramic tableware. Enterprise A sells its products to the “Epicentr” (strategy A1) and “Metro” (strategy A2) hypermarkets, and enterprise B also follows a similar supply line with the respective strategies B1 and B2. Depending on the quality of its products, each of the enterprises can attract a certain percentage of the customers of the competing enterprise. Determine the optimal strategies of the players and the price of the game according to the payment matrix A (percentage of customers attracted or lost):

	A	B	C	D	E
1	**Task 4.** Let enterprises A and B produce ceramic tableware. Enterprise A sells its products to the “Epicentr” (strategy A1) and “Metro” (strategy A2) hypermarkets, and enterprise B also follows a similar supply line with the respective strategies B1 and B2. Depending on the quality of its products, each of the enterprises can attract a certain percentage of the customers of the competing enterprise. Determine the optimal strategies of the players and the price of the game according to the payment matrix A (percentage of customers attracted or lost):		A=	7	-3
2				-4	6
3	**Answwer:**				
4	The answer of the game is the mixed strategies of players A and B.		X=	1/2	1/2
5					
6			Y=	4/9	5/9
7					
8	with the price of the game:		v=	11/2	

Figure 2. *Implementing the solution to problem 4 using MS EXCEL.*

CONCLUSIONS

Game theory is the most widely used method. Many economic processes can be modelled, and solutions can be found only when using game theory. The study shows the possibility of applying game theory to model the consequences of managerial decisions on the economic security of an organisation. The conceptual model defines input and output parameters. The model obtained as a result of solving the game can be used repeatedly to obtain the statistical results necessary for simulation modelling. Given the complexity of modelling security-related processes, this study shows the possibility of applying the game theory method, showing the basic algorithm for model formation and not covering the limitations that inevitably arise when solving specific problems. The use of game theory is impossible without analysing the results by security experts. Such specialists must have competence in the modelled process. The result in the form of prevented losses is understandable to managers at all levels and is acceptable for assessing the effectiveness of security systems.

That is why these methodological recommendations for the discipline ‘Risk Analysis and Management’ are relevant for students to study as independent work.

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Публікується в авторській редакції